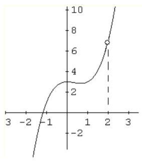
DO NOW

What do you remember about continuity?

Is the following function everywhere continuous?



2.4 Continuity & One-Sided Limits

Continuity at a point on an open interval:

A function is continuous at $x=c$ if there is no interruption in the graph at c. That means - no holes, jumps, or gaps. "Unbroken" at c.

Definition of Continuity:

A function is continuous at c if the following three conditions are met:

1. $f(c)$ is defined
2. $\lim_{x\to c} f(x)$ exists
3. $\lim_{x\to c} f(x) = f(c)$

Continuity on an open interval (a, b) means:

the function is continuous at each point in the interval

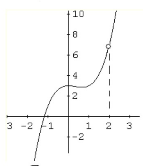
Everywhere continuous -

A function that is continuous on the entire real # line $(-\infty, \infty)$

Two Types of Discontinuity at c:

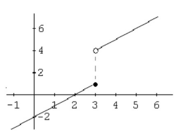
1. Removable - if f can be made continuous by defining (or redefining) $f(c)$.
   Here you will be able to factor and cancel or rationalize the numerator to "Remove" the discontinuity
2. Nonremovable - f cannot be made (or redefined) into a continuous function.

Examples of discontinuity:



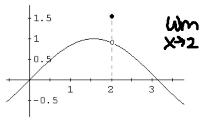
* $f(2)$ is not defined

Removable discontinuity at $x=2$



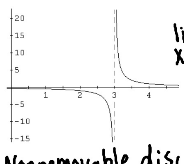
$\lim_{x\to 3} f(x) = \text{DNE}$

Nonremovable discontinuity at $x=3$



$\lim_{x\to 2} f(x) \neq f(2)$

Removable discontinuity at $x=2$



$\lim_{x\to 3} f(x) = \text{DNE}$

Nonremovable discontinuity at $x=3$

One-Sided Limits - limit of a function approaching from the RIGHT OR LEFT

1. Limit from the right $\lim_{x\to c^+}$
2. Limit from the left $\lim_{x\to c^-}$

Theorem - The existence of a limit: If f is a function and c & L are $\mathbb{R}$, then the limit of $f(x)$ as $x\to c$ is L

IF AND ONLY IF

$\lim_{x\to c^-} f(x) = L$ AND $\lim_{x\to c^+} f(x) = L$

Examples:

1. $\lim_{x\to 2} \frac{|x-2|}{x-2}$

$x<2 \quad x-2 \to$ negative

$x=2 \quad x-2=0$

$x>2 \quad x-2 \to$ positive

$\lim_{x\to 2^-} \frac{|x-2|}{x-2}$ $\qquad$ $\lim_{x\to 2^+} \frac{|x-2|}{x-2}$

$\lim_{x\to 2^-} \frac{-(x-2)}{x-2}$ $\qquad$ $\lim_{x\to 2^+} \frac{x-2}{x-2}$

$\lim_{x\to 2^-} \frac{|x-2|}{x-2} = \boxed{-1}$ $\qquad$ $\lim_{x\to 2^+} \frac{|x-2|}{x-2} = \boxed{1}$

$\lim_{x\to 2^-} \neq \lim_{x\to 2^+}$

$\lim_{x\to 2} \frac{|x-2|}{x-2} = \text{DNE}$

2. $\lim_{x\to -3} f(x)$ where $f(x) = \begin{cases} 5 - 2x, & x < -3 \\ x^2 + 2, & x \geq -3 \end{cases}$

left $\lim_{x\to -3^-} f(x)$ $\qquad$ right $\lim_{x\to -3^+} f(x)$

$\lim_{x\to -3^-} (5-2x)$ $\qquad$ $\lim_{x\to -3^+} (x^2+2)$

$5-2(-3)$ $\qquad$ $(-3)^2+2$

$5+6$ $\qquad$ $9+2$

$\boxed{11}$ $\qquad$ $\boxed{11}$

$\boxed{\lim_{x\to -3} f(x) = 11}$

HOMEWORK

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